

# Properties Of Integrals

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## Question 1

Qualification: AP Calculus AB

Areas: Integration, Applications of Differentiation, Limits and Continuity

Subtopics: Properties of Integrals, Fundamental Theorem of Calculus (First), Concavity, Tangents To Curves, Mean Value Theorem, Continuities and Discontinuities, Derivative Tables

Paper: Part B-Non-Calc / Series: 2002 / Difficulty: Hard / Question Number: 6

$x$	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

6. Let  $f$  be a function that is differentiable for all real numbers. The table above gives the values of  $f$  and its derivative  $f'$  for selected points  $x$  in the closed interval  $-1.5 \leq x \leq 1.5$ . The second derivative of  $f$  has the property that  $f''(x) > 0$  for  $-1.5 \leq x \leq 1.5$ .

- (a) Evaluate  $\int_0^{1.5} (3f'(x) + 4) dx$ . Show the work that leads to your answer.
- (b) Write an equation of the line tangent to the graph of  $f$  at the point where  $x = 1$ . Use this line to approximate the value of  $f(1.2)$ . Is this approximation greater than or less than the actual value of  $f(1.2)$ ? Give a reason for your answer.
- (c) Find a positive real number  $r$  having the property that there must exist a value  $c$  with  $0 < c < 0.5$  and  $f''(c) = r$ . Give a reason for your answer.
- (d) Let  $g$  be the function given by  $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \geq 0 \end{cases}$ .

The graph of  $g$  passes through each of the points  $(x, f(x))$  given in the table above. Is it possible that  $f$  and  $g$  are the same function? Give a reason for your answer.

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## Question 2

Qualification: AP Calculus AB

Areas: Limits and Continuity, Integration, Applications of Integration

Subtopics: Continuities and Discontinuities, Calculating Limits Algebraically, Average Value of a Function, Properties of Integrals, Integration Technique – Standard Functions, Differentiability

Paper: Part B-Non-Calc / Series: 2003 / Difficulty: Very Hard / Question Number: 6

6. Let  $f$  be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5-x & \text{for } 3 < x \leq 5. \end{cases}$$

- (a) Is  $f$  continuous at  $x = 3$ ? Explain why or why not.
- (b) Find the average value of  $f(x)$  on the closed interval  $0 \leq x \leq 5$ .
- (c) Suppose the function  $g$  is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ mx + 2 & \text{for } 3 < x \leq 5, \end{cases}$$

where  $k$  and  $m$  are constants. If  $g$  is differentiable at  $x = 3$ , what are the values of  $k$  and  $m$ ?

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